

Quadratic reheating

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Abstract

The reheating process for the inflationary scenario is investigated phenomenologically. The decay of the oscillating massive inflaton field into light bosons is modeled after an out of equilibrium mixture of interacting fluids within the framework of irreversible thermodynamics. Self-consistent, analytic results for the evolution of the main macroscopic magnitudes like temperature and particle number densities are obtained. The models for linear and quadratic decay rates are investigated in the quasiperfect regime. The linear model is shown to reheat very slowly while the quadratic one is shown to yield explosive particle and entropy production. The maximum reheating temperature is reached much faster and its magnitude is comparable with the inflaton mass.

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1 Introduction

During an inflationary stage driven by the potential energy of a classical scalar field, the temperature of the universe was redshifted to almost zero. Thus a mechanism to raise the temperature of the universe at the end of the inflationary stage is required to match this scenario with the standard hot Big Bang cosmology [1, 2].

Currently it is assumed that soon after the end of inflation, the scalar field began to oscillate near the minimum of its effective potential. Decay of this scalar field through particle production and thermalization of its energy with a huge increase in temperature and large entropy production, occurred during the reheating period. It started with violent particle production through parametric resonance, also called the preheating process, and lead to a highly nonequilibrium distribution of the produced particles, subsequently relaxing to an equilibrium state [3, 4, 5].

In this paper we will consider the decay of the massive homogeneous inflaton scalar field ϕ coupled with light bosons χ , as it arises in chaotic inflationary models. A detailed quantum mechanical analysis is quite complicated and involves taking into account nonlinear processes, backreaction effects, rescattering, etc [6]. Further, since particles created during the preheating stage are far from equilibrium, their thermalization is achieved via collisional relaxation. This process of thermalization has not been understood yet from first principles [7]. Here, we follow an alternative description of the physics, and instead of both fields and their interaction we consider an effective, phenomenological model in terms of an out-of-equilibrium mixture of two reacting fluids within the framework of relativistic irreversible thermodynamics [8, 9]. The difference in the equations of state of the fluid components and the simultaneous deviation from the detailed balance in the rate equations for the interfluid reactions give rise to an entropy production effect that manifests itself as a reactive bulk pressure [10]. This two-fluid model can be described in the effective one-fluid picture, based on a single temperature that we identify with the temperature of the reheating process. One of the advantages of this approach is that it is possible to treat selfconsistently the dynamics of geometry and matter [11].

In this phenomenological approach we ignore microscopic details and concentrate on most relevant macroscopic magnitudes like particle number density, temperature, entropy production. The net effect of the interaction term

between both fields is collected in the particle number rate of change, and the finite temperature takes into account the backreaction effect of the produced particles. A phenomenological approach was initially proposed in [11]. The methods used in that paper for the solution of the evolution equations lead to a vanishing bulk viscous contribution to the speed of sound. However their model depends strongly on this contribution so that their procedure of calculation needs to be improved. In this direction we introduce a physically founded perturbative scheme that yields more satisfactory results.

The plan of this paper is as follows. Section 2 introduces the two-fluid magnitudes and section 3 presents main equations of the model in the one-temperature picture. In section 4 we introduce an expansion in powers of a parameter that measures de departure from perfect fluid behavior. This allows us to solve the system of equations order by order. In section 5 the model with a linear ansatz for the decay rate is solved, and we show that it reaches the reheating temperature only for very large times. We propose in section 6 a quadratic ansatz for the decay rate that is better motivated both from microphysical and geometrical point of view. We investigate thoroughly the evolution of the physical magnitudes and we find that the behavior of this model is very interesting. Finally, conclusions are stated in section 7.

2 Two-fluid picture

For the two-fluid model we assume that the energy-momentum tensor T^{ik} splits into two perfect fluid parts,

$$T^{ik} = T_1^{ik} + T_2^{ik} , \quad (1)$$

with $(A = 1, 2)$

$$T_A^{ik} = \rho_A u^i u^k + p_A h^{ik} . \quad (2)$$

ρ_A is the energy density and p_A is the equilibrium pressure of species A . For simplicity we assume that both components share the same 4-velocity u^i . The quantity h^{ik} is the projection tensor $h^{ik} = g^{ik} + u^i u^k$. The particle flow vector N_A^i of species A is defined as

$$N_A^i = n_A u^i , \quad (3)$$

where n_A is the particle number density. We are interested in situations where neither the particle numbers nor the energy-momenta of the components are separately conserved. The balance laws for the particle numbers are

$$N_{A;i}^i = \dot{n}_A + \Theta n_A = n_A \Gamma_A , \quad (4)$$

where $\Theta \equiv u_{;i}^i$ is the fluid expansion and Γ_A is the rate of change of the number of particles of species A . There is particle production for $\Gamma_A > 0$ and particle decay for $\Gamma_A < 0$, respectively. For $\Gamma_A = 0$, we have separate particle number conservation.

Interactions between the fluid components amount to the mutual exchange of energy and momentum. Consequently, there will be no local energy-momentum conservation for the subsystems separately. Only the energy-momentum tensor of the system as a whole is conserved.

Denoting the loss- and source-terms in the separate balances by t_A^i , we write

$$T_{A;k}^{ik} = -t_A^i , \quad (5)$$

implying

$$\dot{\rho}_A + \Theta (\rho_A + p_A) = u_a t_A^a , \quad (6)$$

and

$$(\rho_A + p_A) \dot{u}^a + p_{A,k} h^{ak} = -h_i^a t_A^i . \quad (7)$$

All the considerations to follow will be independent of the specific structure of the t_A^i .

Each component is governed by a separate Gibbs equation

$$T_A ds_A = d\frac{\rho_A}{n_A} + p_A d\frac{1}{n_A} , \quad (8)$$

where s_A is the entropy per particle of species A . Using eqs.(4) and (6) one finds for the time behaviour of the entropy per particle

$$n_A T_A \dot{s}_A = u_a t_A^a - (\rho_A + p_A) \Gamma_A . \quad (9)$$

With nonvanishing source terms in the balances for n_A and ρ_A , the change in the entropy per particle is different from zero in general.

The equations of state of the fluid components are assumed to have the general form

$$p_A = p_A(n_A, T_A) \quad (10)$$

and

$$\rho_A = \rho_A(n_A, T_A) \quad , \quad (11)$$

i.e., particle number densities n_A and temperatures T_A are regarded as the basic thermodynamical variables. The temperatures of the fluids are different in general.

Differentiating relation (11), using the balances (4) and (6) as well as the general relation

$$\frac{\partial \rho_A}{\partial n_A} = \frac{\rho_A + p_A}{n_A} - \frac{T_A}{n_A} \frac{\partial p_A}{\partial T_A} \quad , \quad (12)$$

that follows from the requirement that the entropy is a state function, we find the following expression for the temperature behaviour [12, 13, 14]:

$$\dot{T}_A = -T_A (\Theta - \Gamma_A) \frac{\partial p_A / \partial T_A}{\partial \rho_A / \partial T_A} + \frac{u_a t_A^a - \Gamma_A (\rho_A + p_A)}{\partial \rho_A / \partial T_A} \quad . \quad (13)$$

The entropy flow vector S_A^a is defined by

$$S_A^a = n_A s_A u^a \quad , \quad (14)$$

and the contribution of component A to the entropy production density becomes

$$\begin{aligned} S_{A;a}^a &= n_A s_A \Gamma_A + n_A \dot{s}_A \\ &= \left(s_A - \frac{\rho_A + p_A}{n_A T_A} \right) n_A \Gamma_A + \frac{u_a t_A^a}{T_A} \quad , \end{aligned} \quad (15)$$

where relation (9) has been used.

According to eq.(5) the condition of energy-momentum conservation for the system as a whole,

$$\left(T_1^{ik} + T_2^{ik}\right)_{;k} = 0 , \quad (16)$$

implies

$$t_1^a = -t_2^a . \quad (17)$$

There is no corresponding condition, however, for the particle number balance as a whole. Defining the integral particle number density n as

$$n = n_1 + n_2 , \quad (18)$$

we have

$$\dot{n} + \Theta n = n\Gamma , \quad (19)$$

where

$$n\Gamma = n_1\Gamma_1 + n_2\Gamma_2 . \quad (20)$$

Γ is the rate by which the total particle number n changes.

The entropy per particle is [15]

$$s_A = \frac{\rho_A + p_A}{n_A T_A} - \frac{\mu_A}{T_A} , \quad (21)$$

where μ_A is the chemical potential of species A . Introducing the last expression into eq.(15) yields

$$S_{A;a}^a = -\frac{\mu_A}{T_A} n_A \Gamma_A + \frac{u_a t_A^a}{T_A} . \quad (22)$$

For the total entropy production density

$$S_{;a}^a = S_{1;a}^a + S_{2;a}^a \quad (23)$$

we obtain

$$S_{;a}^a = -\frac{\mu_2}{T_2} n\Gamma - \left(\frac{\mu_1}{T_1} - \frac{\mu_2}{T_2}\right) n_1 \Gamma_1 + \left(\frac{1}{T_1} - \frac{1}{T_2}\right) u_a t_1^a . \quad (24)$$

We assume that the entropy per particle of each of the components is preserved. The particles decay or are produced with a fixed entropy s_A .

This ‘isentropy’ condition amounts to the assumption that the particles at any stage are amenable to a perfect fluid description. When $\dot{s}_A = 0$, we get from (9)

$$u_a t_A^a = (\rho_A + p_A) \Gamma_A , \quad (25)$$

and combining eqs.(17) and (25), one has

$$u_a t_1^a = (\rho_1 + p_1) \Gamma_1 = -u_a t_2^a = -(\rho_2 + p_2) \Gamma_2 , \quad (26)$$

which provides us with a relation between the rates Γ_1 and Γ_2 :

$$\Gamma_2 = -\frac{\rho_1 + p_1}{\rho_2 + p_2} \Gamma_1 . \quad (27)$$

Inserting the last relation into equation (20) yields

$$n\Gamma = n_1 \Gamma_1 h_1 \left(\frac{1}{h_1} - \frac{1}{h_2} \right) . \quad (28)$$

The quantities $h_A \equiv (\rho_A + p_A) / n_A$ are the enthalpies per particle.

With the relations (25) and (28) the entropy production density (24) becomes

$$S_{;a}^a = (\rho_1 + p_1) \left[\frac{n_1 s_1}{\rho_1 + p_1} - \frac{n_2 s_2}{\rho_2 + p_2} \right] \Gamma_1 = n_1 \Gamma_1 h_1 \left[\frac{s_1}{h_1} - \frac{s_2}{h_2} \right] . \quad (29)$$

We emphasize that according to the equations of state (10) and (11) the quantities ρ_1, p_1 and s_1 depend on T_1 , while ρ_2, p_2 and s_2 depend on T_2 . In general, we have $T_1 \neq T_2$.

To complete the two-fluid model we give the thermodynamical properties of both fluids. If we ignore effects associated with particle creation, after inflation the field ϕ oscillates near the point $\phi = 0$ at a frequency given by its mass m . Its oscillation amplitude Φ falls off as t^{-1} and its mean energy density $\rho_\phi = m^2 \Phi^2 / 2$ decreases as a^{-3} , in the same way as pressureless dust [18]. Hence we represent the massive inflaton field as a fluid, named 1, of nonrelativistic particles with mass m , energy density $\rho_1 = \rho_\phi$, pressure $p_1 \ll \rho_1$ and number density $n_1 \simeq \rho_1 / m$. On the other hand, the decay products of the scalar field ϕ are ultrarelativistic for $m \gg m_\chi$, and we model

them as an ideal relativistic fluid of massless particles, with energy density ρ_2 , equilibrium pressure $p_2 = \rho_2/3$ and particle density n_2 . We also assume that both fluids are classical and ideal and that each species remain in thermal equilibrium along the reheating process. That is, we neglect dissipative effects for each of the fluids in comparison with particle production effects.

3 Effective one-temperature picture

We have also the effective one-temperature alternative description of a two-component fluid that is based on a single Gibbs-equation for the system as a whole:

$$Tds = d\frac{\rho}{n} + pd\frac{1}{n} - (\mu_1 - \mu_2) d\frac{n_1}{n} , \quad (30)$$

where p is the equilibrium pressure, ρ is the energy density and s is the entropy per particle. The temperature T is the equilibrium temperature of the whole system and is defined by [16, 17]

$$\rho_1(n_1, T_1) + \rho_2(n_2, T_2) = \rho(n, n_1, T) . \quad (31)$$

Furthermore, we assume that the cosmic fluid as a whole is characterized by the equations of state

$$p = p(n, n_1, T) \quad (32)$$

and

$$\rho = \rho(n, n_1, T) , \quad (33)$$

The temperatures T_1 and T_2 do not appear as variables in the effective one-temperature description. An (approximate) equilibrium for the entire system is assumed to be established through the interactions between the subsystems on the right-hand side of the balances (6) and (7). In this picture the cosmic fluid splits into two effective fluid components whose equations of state are assumed to have the same form as those of the "real" fluids, but share this common temperature. So, these two effective fluids are in a (formal) thermal equilibrium between them while the "real" fluids are not.

If we set $T_1 = T_2 = T$ in (21) and make the splitting for the energy density of the system

$$\rho(T) = \rho_1(T) + \rho_2(T) \quad (34)$$

and for its equilibrium pressure

$$p(T) = p_1(T) + p_2(T), \quad (35)$$

the description based on relation (30) is consistent with the description relying on the relations (8) for $ns(T) = n_1s_1(T) + n_2s_2(T)$. As long as the pressures are those for classical gases, $p_A = n_AT$, the equilibrium pressure p of the system as a whole depends on $n = n_1 + n_2$ only and the separate dependence on n_1 on the right-hand side of eq.(32) may be omitted.

For components out of equilibrium, even when $\Gamma = \Gamma_1 = \Gamma_2 = 0$, the total pressure is generally different to the sum of the partial pressures. In this case we have [17]

$$p_1(n_1, T_1) + p_2(n_2, T_2) = p(n, T) + \pi_d. \quad (36)$$

instead of (35). Here π_d is the viscous pressure arising from differential temperature variation rate between both fluids [17]. From eq.(13) the cooling rate \dot{T}_1/T_1 is different from \dot{T}_2/T_2 even for $\Gamma_1 = \Gamma_2 = 0$ if the subsystems are governed by different equations of state. The expansion of the Universe tends to increase the difference between T_1 and T_2 .

On the other hand, deviations from detailed balance, i.e., $\Gamma_A \neq 0$, leading to $\Gamma \neq 0$ in general, generate an effective ‘reactive’ bulk pressure π . For the corresponding energy-momentum tensor of the system as a whole we write

$$T^{ik} = \rho u^i u^k + (p + \pi) h^{ik}. \quad (37)$$

The reactive bulk pressure is determined by the consistency of the expression for the entropy production density obtained in the two-temperature picture (29) and the expression obtained in the one-temperature picture

$$S^\mu_{;\mu} = sn\Gamma + n\dot{s} \quad (38)$$

The last one arises from the entropy flow vector $S^\mu = snu^\mu$ [8], where s is the entropy per particle and u^μ is the four-velocity of the fluid. It was estimated that π_d is one order of magnitude smaller than π [10].

From the Gibbs-equation (30) one finds for the change in the entropy per particle

$$n\dot{s} = -\frac{\Theta}{T}\pi - \frac{\rho+p}{T}\Gamma - \frac{n_1 n_2}{n} \left(\frac{\mu_1 - \mu_2}{T} \right) (\Gamma_1 - \Gamma_2) . \quad (39)$$

Even for $\dot{s}_1 = \dot{s}_2 = 0$ we have $\dot{s} \neq 0$ in general. However, during the preheating stage, particle production dominates over per particle entropy change as the main entropy production source.

The equilibrium temperature of the system T is the key magnitude of our model. On the one hand we identified it with the reheating temperature, and on the other hand its definition (31) provides a mean to account for backreaction effects of the created relativistic particles on their massive counterparts. Its evolution law is given by [10]

$$\frac{\dot{T}}{T} = (\Gamma - \Theta) \frac{\partial p / \partial T}{\partial \rho / \partial T} - \frac{\Theta \pi}{T \partial \rho / \partial T} \quad (40)$$

The Einstein field equations for a spatially-flat Robertson-Walker space-time are

$$3H^2 = \kappa \rho , \quad (41)$$

$$\dot{H} = -\frac{\kappa}{2} (\rho + p + \pi) , \quad (42)$$

where κ is Einstein's gravitational constant and $H = \dot{a}/a$, a is the cosmic scale factor. We use units such that $c = 1$, $k_B = 1$ and $\hbar = 1$, then $\kappa = 8\pi/M_P^2$, where M_P is the Planck mass. Finally a dot denotes derivative with respect to comoving time. Conservation of (37) leads to

$$\dot{\rho} + 3H(\gamma\rho + \pi) = 0 \quad (43)$$

where

$$\gamma(t) \equiv 1 + \frac{p(t)}{\rho(t)} \quad (44)$$

is the time dependent polytropic index. Using (41) and (42) we get

$$\kappa\pi = -3\gamma H^2 - 2\dot{H} \quad (45)$$

When $m \gg T$, the equations of state for the nonrelativistic fluid 1 are:

$$\rho_1 = n_1 m + \frac{3}{2} n_1 T, \quad p_1 = n_1 T \quad (46)$$

while for the relativistic fluid 2 are:

$$\rho_2 = 3n_2 T, \quad p_2 = n_2 T \quad (47)$$

For these fluids we have

$$\gamma = 1 + \frac{nT}{n_1 m + \left(\frac{3}{2} n_1 + 3n_2\right) T} \quad (48)$$

To calculate the evolution of the temperature we need also Γ . Using (28) and the equations of state (46) and (47), we obtain

$$\Gamma = \frac{n_1 m}{4nT} Q \quad (49)$$

where the decay rate $Q = |\Gamma_1|$ is an input to the model that must be chosen from a fundamental microphysical theory or from phenomenological considerations.

4 The quasiperfect expansion

We assume that the viscous effects, are small. If τ is the mean interaction time of the particles of the fluid, we have that $\nu = (\tau H)^{-1}$ is the number of interactions in an expansion time. Perfect fluid behavior occurs in the limit $\nu \rightarrow \infty$. Small departures from this behavior occur for large ν , and a consistent hydrodynamical description of the fluids requires $\nu > 1$. Thus we are lead to assume that τH is small and we propose the following "quasiperfect" expansion in powers of ν^{-1}

$$H = H_0 \left(1 + \frac{h_1}{\nu} + \dots \right) \quad (50)$$

$$\pi = \frac{\pi_1}{\nu} + \frac{\pi_2}{\nu^2} + \dots \quad (51)$$

where π_i , will be fixed by the thermodynamical theory adopted. Inserting (50) and (51) in (45), we find the equations that determine the coefficients of the expansion up to first order in ν^{-1}

$$\dot{H}_0 + (3/2)\gamma H_0^2 = 0 \quad (52)$$

$$\dot{h}_1 = -\frac{\pi_1}{2H_0} \quad (53)$$

where we have assumed that $\dot{\tau} \ll \tau H$. Solving these equations we get

$$H_0(t) = \frac{2}{3 \int dt \gamma(t)} \quad (54)$$

and

$$H = H_0 \left(1 - \frac{\kappa}{2\nu} \int dt \frac{\pi_1}{H_0} + \dots \right) \quad (55)$$

In the particular case we choose the truncated transport equation of Causal Irreversible Thermodynamics for the bulk viscosity pressure

$$\pi + \tau \dot{\pi} = -3\zeta H. \quad (56)$$

where we identify τ with the relaxation time, the bulk viscosity coefficient ζ is given by [19]

$$\frac{\zeta}{\tau} = c_b^2(\rho + p), \quad (57)$$

c_b is the bulk viscous contribution to the speed of sound v , $v^2 = c_s^2 + c_b^2 \leq 1$, and c_s is the adiabatic sound speed, we find

$$\kappa\pi \approx -\frac{9}{\nu} c_b^2 \gamma H_0^2 \quad (58)$$

Also it can be easily seen that H_0 is an exact solution of equations (41), (42), (44), (56) and (57) provided $c_b = 0$.

To get an estimation of the physical parameters in the quasiperfect regime it is enough to keep calculations at zero order in ν^{-1} and we can neglect the viscous terms in (43), (40) and (48). In this order of approximation the results are independent of the form of the transport equation. Nevertheless

this approach will allow us to give a reasonable description of the reheating process. Then (43) becomes

$$\dot{\rho} + 3H_0\gamma\rho \approx 0 \quad (59)$$

whose solution is

$$\rho(t) \approx \frac{\rho_0}{(\int dt\gamma(t))^2} \quad (60)$$

where ρ_0 is a positive integration constant. Also (40) becomes

$$\frac{\dot{T}}{T} \approx \frac{2n}{3(n_1 + 2n_2)} \left(\frac{n_1 m}{4nT} Q - 3H_0 \right) \quad (61)$$

Then, when the decay rate Q is large enough, it may overcompensate the adiabatic term during the initial ‘preheating’ stage and make the temperature rise.

5 Linear reheating

Following [11] we assume first a linear ansatz $Q = \beta H$, with an adimensional constant $\beta > 0$.

To solve the coupled system of equations (19), (4), (61) and (48) for the evolution for n , n_1 , n_2 , T and γ , we use an iterative scheme that starts from a fully nonrelativistic stage: $n_1 = n$, $\gamma = 1$. In this regime

$$H_0 \approx \frac{2}{3t} \quad (62)$$

Then (61) reduces to

$$\frac{\dot{T}}{T} \simeq \frac{4}{9t} \left(\frac{m\beta}{4T} - 3 \right) \quad (63)$$

whose solution is

$$T(t) = T_1 \left[1 - \left(\frac{t_0}{t} \right)^{4/3} \right] \quad (64)$$

where t_0 is an arbitrary integration constant with dimension of time, and $T_1 = m\beta/12$. In order to describe the reheating scenario we need to choose $t_0 > 0$. As in the previous inflationary stage the Universe "supercools" [1], we assume that the inflation period ends at $t \approx t_0$ when $T \approx 0$, and we take solution (64) for $t > t_0$. Then the temperature grows monotonically and approaches asymptotically its maximum value T_1 , which we could call the reheating temperature of the linear model, for $t \rightarrow \infty$. We see from (64) that the temperature rises quite slowly. It takes a time over $30t_0$ for the temperature to approach at 1% of T_1 . For this reason, we will not pursue further the consequences of the linear ansatz and in a following section we will propose an improved ansatz for Q which leads a much faster rise in temperature.

6 Quadratic reheating

In the previous section, we have seen that a linear ansatz for Q leads to a very slow process of reheating. A microscopic interaction term like $g\phi^2\chi^2$, suggests an effective quadratic decay rate $\Gamma \sim \Phi^2 \sim t^{-2}$. Here we look for an improved ansatz that takes into account the scalar nature of Γ . In this direction we propose $Q = \bar{\beta}R$, where $R = 6(\dot{H} + 2H^2)$ is the curvature scalar and $\bar{\beta}$ is a positive constant with dimension of time.

At zero order in ν^{-1} we obtain

$$Q = 6\bar{\beta} \left(2 - \frac{3\gamma}{2}\right) H_0^2 \equiv 6\tau_1 H_0^2 \quad (65)$$

This expression shows that $Q \geq 0$ for $1 \leq \gamma \leq 4/3$ and vanishes in a radiation dominated era, when Φ is very small. Inserting (65) in (49) and using (61) we obtain at zero order in ν^{-1}

$$\frac{\dot{T}}{T} \simeq \frac{nH_0}{n_1 + 2n_2} \left(\frac{n_1 m \tau_1}{nT} H_0 - 2 \right) \quad (66)$$

and we solve it using the iterative scheme.

6.1 Nonrelativistic regime

In this regime when the fluid is dominated by massive particles, equation (66) becomes

$$\dot{T} + 2H_0 T \simeq m\tau_1 H_0^2 \quad (67)$$

whose general solution is

$$T(t) = \frac{4m\tau_1}{3t} \left[1 - \left(\frac{t_0}{t} \right)^{1/3} \right] \quad (68)$$

As in the linear model, the reheating scenario corresponds to $t_0 > 0$, and we take the solution (68) for $t > t_0$. This solution also starts from $T = 0$, but this time it rises violently to a maximum temperature of reheating

$$T_r = \frac{9\tau_1}{64t_0} m \quad (69)$$

in a period $t_r - t_0 = 37t_0/27$, where t_r is the time when maximum temperature occurs. Assuming that inflation ends at $t_0 = \delta/m$, where δ is a numeric constant, this rising period is $1.37\delta/m$. This is a time of the same order as the period of oscillation of the inflaton field $2\pi/m$. After that, the temperature begins to fall. Though these calculations are carried out in the approximation $T \ll m$, and this implies that (69) is strictly valid only for $\tau_1 \ll t_0$, it also suggests that T_r may be of order m when τ_1 is large enough.

Inserting (62) and (65) in (4) and solving the resulting equation for the number density of massive particles we find

$$n_1(t) \simeq n_{10} \frac{e^{8\tau_1/(3t)}}{t^2} \quad (70)$$

where n_{10} is a positive integration constant, so that $n_1(t)$ is a decreasing function.

Using (19), (49), (68) and (70), we find the total number particle density in the first departure from the nonrelativistic initial state

$$n(t) \simeq \frac{n_0}{t^2} \left[1 + \frac{n_{10}}{2n_0} \int dx \frac{e^{8\tau_1/(3t_0x)}}{x - x^{2/3}} \right] \quad (71)$$

where $x = t/t_0$, and n_0 is a positive constant. Assuming that the reheating temperature is much smaller than m we obtain the approximated expression

$$n(t) \simeq \frac{n_0}{t^2} \left[1 + \frac{3n_{10}}{2n_0} \ln(x^{1/3} - 1) \right] \quad (72)$$

The expression (70) for the density of nonrelativistic particles is a decreasing function, while the expression (72) for the total number density has a peak. The consistency requirement $n_1 \leq n$ is satisfied after the time t_p when $n_1(t_p) = n(t_p) \equiv n_p$. This time is given by

$$x_p = \left[1 + \exp\left(\frac{2(\alpha - 1)}{3\alpha}\right) \right]^3 \quad (73)$$

where $\alpha = n_{10}/n_0$, and marks in our model the starting point of the preheating stage. We demand the temperature

$$T(t_p) \approx 4 \left(\frac{4}{3}\right)^3 (x_p^{1/3} - 1) T_r \quad (74)$$

to be very low and this occurs when x_p is close to 1, so that $\alpha \ll 1$. This condition corresponds to an explosive production of light bosons. In effect, from (18), (72) and (70) we get

$$n_2(t) \approx \frac{3n_{10}}{2t^2} \ln\left(\frac{x^{1/3} - 1}{x_p^{1/3} - 1}\right) \quad (75)$$

and just after t_p this particle density grows linearly with an exponentially large slope

$$n_2(t) \approx \frac{n_{10} e^{2/(3\alpha)}}{2t_0^2} (x - x_p), \quad x - x_p \ll 1 \quad (76)$$

reaching a large peak density n_p/α in a time $x_m \approx 1 + 3\alpha/4$. After that, both n and n_2 begin to fall rapidly as the effect of expansion dominates over particle production.

In the regime of large particle production we can neglect in (38) the change in s , so that the entropy production density is approximately given by

$$S_{;a}^a \approx sn\Gamma = \frac{sn_{10} e^{8\tau_1/(3t)}}{2t^2 (t - t_0^{1/3} t^{2/3})} \quad (77)$$

and we obtain the following lower bound for the entropy density

$$S > \frac{sn_{10}}{2t_0^2} \int_{x_p}^{x_r} \frac{dx}{x^2 (x - x^{2/3})} \approx -\frac{3sn_{10}}{2t_0^2} \ln (x_p^{1/3} - 1) \approx \frac{sn_0}{t_0^2} \quad (78)$$

Thus we find that the generated entropy per massive particle $S/n(t_p) > s/\alpha$ is very high.

6.2 Intermediate regime

Here $\rho_1 \approx \rho_2$, i.e. $n_1 m \approx 3n_2 T$ so that $n_2 \gg n_1$ and $\gamma \approx \frac{7}{6}$. At the microscopic level this corresponds to the regime when the backreaction of the produced particles ceases to be negligible. We can estimate the time t_i when this regime begins from the equation $\rho(t_i) = 2\rho_1(t_i)$, using (46), (60), (68) and (70). Therefore t_i becomes a function of $K \equiv m\tau_1/\delta$ and the intermediate regime is reached provided $K > 3 \ln 2/8$. For lower values of K particle production rate is not large enough to compensate the faster decrease of relativistic matter energy density due to cosmic expansion. On the other hand, K cannot be much larger than this lower bound if $T_r \ll m$. For instance, we get $x_i \approx 41$ and $T_r/m \approx 0.038$ for $K = 0.27$. We show in Fig. 1 the evolution of ρ_1 and ρ_2 for this value of K , from fully nonrelativistic to intermediate regime.

In this intermediate regime holds

$$H_0(t) \approx \frac{4}{7t} \quad (79)$$

$$T(t) \approx T_0^{(i)} \frac{e^{-24\tau_1/(49t)}}{t^{4/7}} \quad (80)$$

$$n_1(t) \approx n_{10}^{(i)} \frac{e^{96\tau_1/(49t)}}{t^{12/7}} \quad (81)$$

$$n(t) \approx n_0^{(i)} \frac{e^{-72\tau_1/(49t)}}{t^{12/7}} \quad (82)$$

where $T_0^{(i)}$, $n_{10}^{(i)}$ and $n_0^{(i)}$ are positive integration constants fixed by matching at t_i expressions (80), (81) and (82) with their nonrelativistic counterparts (68), (70) and (71). After the initial stage of large ultrarelativistic particle

creation in the nonrelativistic regime the nonrelativistic particle decay process slows down. All particle densities and the temperature decrease, though the dilution and cooling rates are smaller because of the slower cosmic expansion. The whole evolution of the temperature and particle number densities along the nonrelativistic and intermediate regimes is plotted in Figs. 2-4 for $K = 0.27$.

6.3 Ultra-relativistic regime

In this last stage of reheating, the energy density becomes dominated by the radiation fluid, i.e. $n_2 T \gg n_1 m$, $\gamma \approx \frac{4}{3}$. Therefore we have

$$H_0 \approx \frac{1}{2t} \quad (83)$$

$$T(t) \approx \frac{T_0^{(u)}}{t^{1/2}} \quad (84)$$

$$n(t) \approx \frac{n_0^{(u)}}{t^{3/2}}, \quad n_1(t) \approx \frac{n_{10}^{(u)}}{t^{3/2}} \quad (85)$$

where $T_0^{(u)}$, $n_0^{(u)}$ and $n_{10}^{(u)}$ are positive integration constants. This stage marks the end of reheating and the cosmic medium approaches a perfect, relativistic fluid with vanishing viscosity and particle production. The temperature and particle densities continue falling at about the adiabatic rate. This is smaller than the intermediate stage rate as the radiation-dominated universe expands even slower. A small remnant of nonrelativistic particles may remain.

7 Conclusions

With the aim of understanding the process of reheating we have developed a phenomenological model of two interacting fluids. In this model the oscillating inflaton field is modeled after a nonrelativistic fluid of massive particles that decay into an ultrarelativistic fluid of massless particles.

We have carried out a “quasiperfect” perturbative expansion to solve the Einstein equations. This corresponds to a small viscous pressure regime. To

the lowest order in this expansion, the thermodynamical magnitudes do not depend on the transport equation for the bulk viscosity.

A particle decay rate, linear in the expansion rate, does not yield a suitable model as the maximum reheating temperature is reached only for very long times. However an alternative quadratic ansatz for the decay rate gives a good behavior of the thermodynamical magnitudes. The last one was investigated in detail along its evolution from the initial nonrelativistic stage to the final domination by ultrarelativistic particles. The equilibrium temperature of the whole system and the produced particle number density rise violently in a time comparable to the oscillation period of the inflaton field. Also, large amounts of entropy are produced in this initial stage of reheating. The time when the energy density of the decay products becomes comparable with the nonrelativistic particles, is similar to the microscopic calculations for the time when backreaction effects become important.

In a future paper we will investigate phenomenological reheating models far from the perfect fluid regime.

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Figure Captions

Figure 1. Plot of the equilibrium energy densities of nonrelativistic and relativistic particles, ρ_1 and ρ_2 in units of $\rho_0 = \rho(t_0)$ vs. adimensional time x , from $x = 1$ to $x \simeq x_i$, for $K = 0.27$. In this case $x_i \simeq 41$.

Figure 2. Plot of the equilibrium temperature T in units of inflaton mass m vs. adimensional time x along the nonrelativistic and intermediate regimes, for $K = 0.27$.

Figure 3. Plot of the nonrelativistic particle number density n_1 in units of the initial density n_p vs. adimensional time x along the nonrelativistic and intermediate regimes, for $K = 0.27$.

Figure 4. Plot of the relativistic particle number density n_2 in units of the initial density n_p vs. adimensional time x along the nonrelativistic and intermediate regimes, for $\alpha = 0.1$ and $K = 0.27$. The inserted plot shows a detail of the evolution of n_2 about its peak.







